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Energy cost and efficiency of riding aerodynamic bicycles

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Summary. Traction resistance (R_t) was determined by towing two cyclists in fully dropped posture on bicycles with an aerodynamic frame with lenticular wheels (AL), an aerodynamic frame with traditional wheels (AT), or a traditional frame with lenticular wheels (TL) in calm air on a flat wooden track at constant speed (8.6–14.6 $m \cdot s^{-1}$). Under all experimental conditions, R_t increased linearly with the square of air velocity (v_a^2); r^2 equal to greater than 0.89. The constant $k = \Delta R_t / \Delta v_a^2$ was about 15% lower for AL and AT (0.157 and 0.155 $N \cdot s^2 \cdot m^{-2}$) than for TL bicycles (0.184 $N \cdot s^2 \cdot m^{-2}$). These data show firstly, that in terms of mechanical energy savings, the role of lenticular wheels is negligible and, secondly, that for TL bicycles, the value of k was essentially equal to that found by others for bicycles with a traditional frame and traditional wheels (TT). The energy cost of cycling per unit distance (C_c, $J \cdot m^{-1}$) was also measured for AT and TT bicycles from the ratio of the O_2 consumption above resting to speed, in the speed range from 4.7 to 11.1 $m \cdot s^{-1}$.The C_c also increased linearly with v_a^2, as described by: $C_c = 30.8 + 0.558\ v_a^2$ and $C_c = 29.6 + 0.606\ v_a^2$ for AT and TT bicycles. Thus from our study it would seem that AT bicycles are only about 5% more economical than TT at 12.5 $m \cdot s^{-1}$ the economy tending to increase slightly with the speed. Assuming a rolling coefficient equal to that observed by others in similar conditions, the mechanical efficiency was about 10% lower for aerodynamic than for conventional bicycles, amounting to about 22% and 25% at a speed of 12.5 $m \cdot s^{-1}$. From these data it was possible to calculate that the performance improvement when riding aerodynamic bicycles, all other things being equal, ought to be about 3%. This compares favourably with the increase of about 4% observed in world record speeds (over distances from 1 to 20 km) after the adoption of the new bicycles.

Key words: Cycling – Tractional resistance – Air resistance – Rolling resistance – Energy cost – Mechanical efficiency – Body size

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Introduction

In track cycling, the relationship between the total force opposing motion (traction resistance (R_t) and air velocity (v_a) squared can be expected to be described by:

$$R_t = R_r + k \times v_a^2 \tag{1}$$

where R_r is the rolling resistance and k is a constant which depends on the air density, on the area projected (A), on the frontal plane and on the dimensionless drag coefficient (di Prampero 1986; Pugh 1973, 1974). From the experimentally determined values of R_r and k, di Prampero et al. (1979) have derived a general equation of motion for constant speed cycling on conventional racing bicycles in the dropped posture. This made possible the calculation of the mechanical output and the energy turnover as a function of air and ground speeds, air density, body size of the cyclist, and the incline of the terrain.

During the last 6 years, following the introduction of newly designed frames and wheels, the aerodynamics of cycling has been substantially improved, leading to a significant increase of the record speeds of classical track distances. The aim of this study was to investigate the aerodynamic characteristics of cycling with the new frames and to derive the relationship between mechanical output and speed. In addition, the oxygen consumption ($\dot{V}O_2$) during constant speed cycling in the dropped posture was also measured, over a speed range compatible with fully aerobic conditions.

Methods

The experiments were performed on two amateur cyclists riding an "aerodynamic" bicycle at the Vigorelli velodrome, 150 m above sea level (Milan, Italy) in calm air [temperature (T) = 277–284 K; barometric pressure (100.4–101.3 kpa) = 753–760 mmHg]. Two series of experiments were performed; in both cases the frame of the bicycle was aerodynamic. The wheels, however, were lenticular for one series and traditional for the other. In addition, a limited number of measurements were also taken with

Table 1: Main anthropometric characteristics of the two subjects

Subject	Body mass kg	Height cm	Body surface area m^2	Frontal area m^2	$\dot{V}O_{2max}$ $l \cdot min^{-1}$
1	75.0	184	1.98	0.393	5.25
2	71.0	186	1.94	0.395	5.16

$\dot{V}O_{2max}$, Maximal oxygen consumption (resting value included)

Table 2. Dimensions of bicycles and wheels

	Aerodynamic	Traditional
Front wheel diameter (cm)	66.0	70.0
Rear wheel diameter (cm)	71.0	70.0
Handle bar height (cm)	75.0	95.0
Saddle heigth (cm) (average)	100.5	100.0
Tyre pressure ($N \cdot cm^{-2}$)	100–110	100–110
Total mass (kg)	14.0	11.6

the two subjects riding a traditionally framed cycle with lenticular wheels. The subjects' characteristics, and frame and wheels dimensions, are given in Tables 1 and 2.

The R_t was determined towing the subjects by motorcycle at constant speed (from 8.6 to 14.6 $m \cdot s^{-1}$) in different trials. The two subjects, in fully dropped posture, kept pedalling at 60–100 revolutions$\cdot min^{-1}$ without a transmission chain to reproduce the air turbulence induced by their moving legs during actual cycling. The average speed of each lap was monitored by means of photocells placed at the finishing line of the track. The towing force was measured using a load cell (50 N full scale) mounted in series on a nylon towing cable of 3.0-mm diameter and 10-m length, a distance which minimized the air turbulence caused by the moving motorbike. The power for the load cell was supplied by a car battery (12 V, d.c.) via a carrier amplifier, and the force signal was fed to an analogue tape recorder (TEAC R-61, Japan). The battery, amplifier and recorder were carried on a holder mounted on the rear part of the chassis of the motorbike and were connected to the load cell by means of light wires attached by adhesive tape to the towing cable. The load cell was calibrated imposing known forces before and after each experimental run. Each subject performed several trials at constant speeds about 5 $km \cdot h^{-1}$ (1.7 $m \cdot s^{-1}$) apart in increasing order.

The A of the subjects riding the aerodynamic bicycle were obtained according to the method suggested by Swain et al. (1987). Photographs were taken of the subjects on the bicycle in a racing posture with a rectangular surface of known area at their side. The outlines of the subjects and of the reference surface were then traced on paper, cut out and weighed. The A were finally obtained comparing the masses of the pictures of the cyclists plus cycle to the reference surface. Body surface area (A_D) was calculated according to DuBois and Dubois (1915).

The R_t was determined over several (4–6) laps at each speed. For each lap, its average value was obtained by integrating numerically the signal of the load cell from the initial to the final point and dividing the so-calculated value by the duration of the same lap. The value of R_t at each speed was finally obtained from the average of the various laps. To evaluate the repeatability of the towing method, at six speeds (8.6, 12.4, 13.2, 13.4, 14.3 and 14.5 $m \cdot s^{-1}$) R_t was measured twice. No statistically significant difference between the two paired sets of data was observed applying the Wilcoxon matched-paired test for non parametric data ($P > 0.10$).

Table 3. The biomechanic (B) and energy (E) characteristics of the frame — wheel combinations investigated are indicated together with the equations, figures and table which report the data in the text

	B		E	
AL	Eq. 2	Fig. 1a		
AT	Eq. 3	Fig. 1b	Eq. 4	Fig. 2a
TL	Table 4			
TT			Eq. 5	Fig. 2b

AL, Aerodynamic frame – lenticular wheels; AT, aerodynamic frame – traditional wheels; TL, traditional frame – lenticular wheels; TT, traditional frame – traditional wheels

In the range of speeds from 4.7 to 11.1 $m \cdot s^{-1}$, the steady-state $\dot{V}O_2$ was also measured in a separate set of experiments as follows. The subject wore a noseclip and exhaled, via a light weight valve system and standard expiratory hose of 4.0-cm inner diameter and 2.5-m length, into a 100-l Douglas bag. This was placed on a motorcycle which followed the cyclist. The respiratory hose was supported by an operator who rode the motorcycle as a passenger. The subject was asked to pedal at constant speed, monitored by the photocells, and, after about 3 to 4 min, the operator initiated (and concluded) the air collection by turning on (and off) the two-way valve of the Douglas bag. The valve automatically activated a stopwatch. Gas collection times ranged from 45 to 62 s. After the experiment, the expired air composition and volume were assessed using a paramagnetic O_2 analiser (Sybron, Taylor, Great Britain), an infrared CO_2 meter (BINOS 1, Leybold-Heraeus, Germany) and a dry gas meter (S.I.M., BRUNT, Italy) and $\dot{V}O_2$ (standard temperature and pressure, dry, STPD) was calculated according to standard open circuit procedures. Steady-state $\dot{V}O_2$ ($ml \cdot s^{-1}$) above resting (assumed = 0.06 $ml \cdot kg^{-1} \cdot s^{-1}$ = 3.6 $ml \cdot kg^{-1} \cdot min^{-1}$) was then divided by the speed ($m \cdot s^{-1}$) and multiplied by the energy equivalent of O_2 (1 ml O_2, STPD = 20.9 J at RQ = 0.96) to yield the energy cost of cycling (C_c, $J \cdot m^{-1}$). The gas exchange measurements were performed with the subjects riding the aerodynamic frame with traditional wheels (AT) or the traditional frame with traditional wheels (TT) combination. The frames wheel combinations investigated are indicated in Table 3 together with the equations, figures and tables where the corresponding data are summarized.

The significance of the difference between the slopes of the linear regression was evaluated by means of the method of comparison of two regression lines of the first kind (Geigy Scientific Tables, 1982).

Results

The overall R_t at all investigated speeds, for the aerodynamic frame with lenticular wheel (AL) and AT bicycles is plotted in Fig. 1a and b as a function of v_a^2. Least squares linear interpolation of the data of Fig. 1a and b yielded:

$$R_t = 0.35 + 0.157 \times v_a^2 \quad (2)$$
$(r^2 = 0.95;\ n = 40)$

and

$$R_t = 2.43 + 0.155 \times v_a^2 \quad (3)$$
$(r^2 = 0.96;\ n = 19)$

where R_t is in newtons and v_a is in metres per second. These data showed that R_t increased as a quadratic

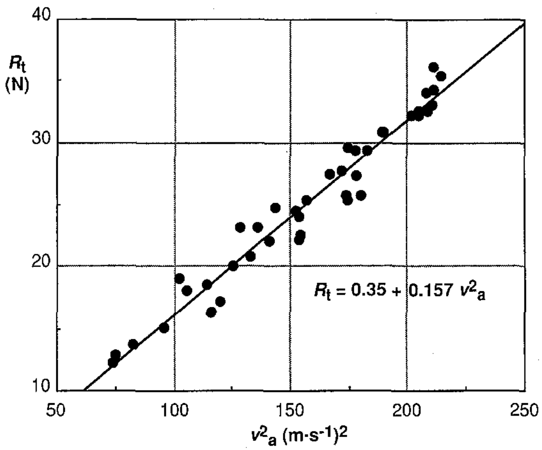

Fig. 1a. Tractional resistance (R_t) as a function of the air velocity squared (v_a^2, $m^2 \cdot s^{-2}$) — aerodynamic frame, lenticular wheels

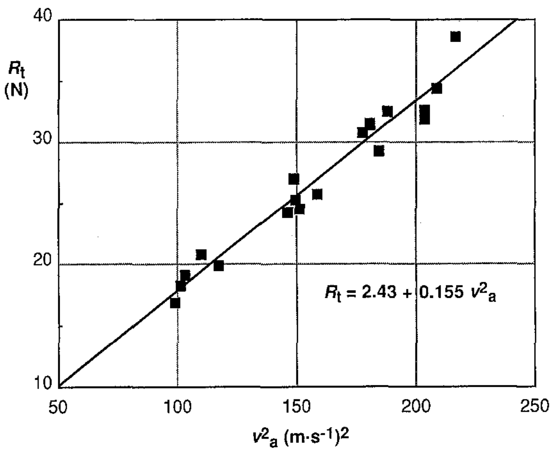

Fig. 1b. Tractional resistance (R_t) as a function of the air velocity squared (v_a^2, $m^2 \cdot s^{-2}$) — aerodynamic frame, traditional wheels

function of v_a and that the slopes of the two functions were not significantly different ($P>0.2$).

In the course of the R_t measurements on the bicycles with traditional frame and lenticular wheels (TL) bicycles, unfortunately, the zero force value was not appropriately calibrated, so that we are not in a position to determine absolute R_t values. However, since the gain of the cell was kept constant throughout the experiment, we could calculate the changes of R_t induced by the changes of speed and the individual coefficients thus obtained $(R_{t_i} - R_{t_0})/(v_{a,i}^2 - v_{a,0}^2)$ are reported in Table 4. The average coefficient, which is a measure of the slope of a plot of R_t versus v_a^2, (see Fig. 1a, b and Eqs. 2, 3) amounted on average to 0.1835 (SD 0.049) $N \cdot s^2 \cdot m^{-2}$. In any case, regardless of the absolute value of R_t, the r^2 of a linear regression of R_t versus v_a^2 was 0.89.

In the range of speeds investigated, the $\dot{V}O_2$ above resting increased from $0.75\ l \cdot min^{-1}$ at a speed of about $5.5\ m \cdot s^{-1}$ to $3.5\ l \cdot min^{-1}$ for speeds of approximately $11.0\ m \cdot s^{-1}$, i.e. well below the maximal $\dot{V}O_2$ of the subjects which amounted on average to

Table 4. $(R_{t_i} - R_{t_0})/(v_{a,i}^2 - v_{a,0}^2)$ coefficients for bicycles with traditional frames with lenticular wheels, where $v_{a,0}$ was equal to $12.7\ m \cdot s^{-1}$

v_a ($m \cdot s^{-1}$)	$R_{t_i} - R_{t_0}$ (N)	$v_{a,i}^2 - v_{a,0}^2$ ($m^2 \cdot s^{-2}$)	$(R_{t_i} - R_{t_0})/(v_{a,i}^2 - v_{a,0}^2)$ ($N \cdot s^2 \cdot m^{-2}$)
12.9	1.0	5.12	0.195
13.3	1.5	15.60	0.096
13.2	2.9	12.95	0.224
13.4	2.3	18.27	0.126
13.4	4.6	18.20	0.252
13.6	5.4	23.67	0.228
13.7	3.1	26.40	0.117
14.0	6.7	34.71	0.193
14.1	8.3	37.52	0.221
14.3	9.0	43.20	0.208
14.5	7.7	48.96	0.157
14.6	9.6	51.87	0.185
		Mean	0.1835
		SD	0.049

v_a, Air velocity; R_{t_i}, R_{t_0}, individual traction resistances induced by changes air velocity $v_{a,i}^2$, $v_{a,0}^2$

$5.2\ l \cdot min^{-1}$ (see Table 1). This rules out the possibility of any substantial anaerobic contribution to the overall energy requirement.

The C_c is plotted in Fig. 2a and b as a function of v_a^2 for AT and TT bicycles. Similarly to R_t, C_c also increased as a quadratic function of the speed; indeed, linear regression of the data of Fig. 2a and b yielded:

$$C_c = 30.8 + 0.558 \times v_a^2 \qquad (4)$$
$(r^2 = 0.76;\ n = 15)$

$$C_c = 29.6 + 0.606 \times v_a^2 \qquad (5)$$
$(r^2 = 0.86;\ n = 14)$

where C_c is in joules per metre and v_a in metres per second. The two slopes were not significantly different ($P>0.2$).

Discussion and data analysis

Mechanical power output

As expected on the theoretical grounds (see Pugh 1973) and as has been previously found by others (Pugh 1974; di Prampero et al. 1979), during constant speed cycling R_t increased linearly with v_a^2 (Fig. 1a, b; Eqs. 2, 3; Table 4). The constant $k = \Delta R_t / \Delta v_a^2$ is a quantitative index of the aerodynamic characteristics of the bicycle-cyclist combination; it amounted to 0.157, 0.155 and to 0.184 $N \cdot s^2 \cdot m^{-2}$ for AL, AT or TL, respectively. The values of k for AL and AT bicycles are close and not significantly different ($P>0.2$, see Fig. 1a, b; Eqs. 2, 3), showing that the effects of the lenticular wheels on the aerodynamics of cycling were very minor indeed. As such they will be ignored.

The value of k obtained in this study for traditional frames ($k = 0.184\ N \cdot s^2 \cdot m^{-2}$, see Table 4) was equal to that obtained by di Prampero et al. (1979) and by Pugh

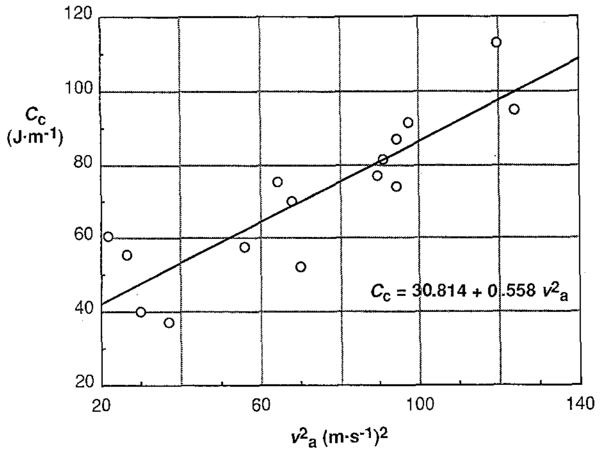

Fig. 2a. Energy cost of cycling for unit distance (C_c, J·m^{-1}) as a function of the square of air velocity (v_a^2, m^2·s^{-2}) — aerodynamic frame, traditional wheels

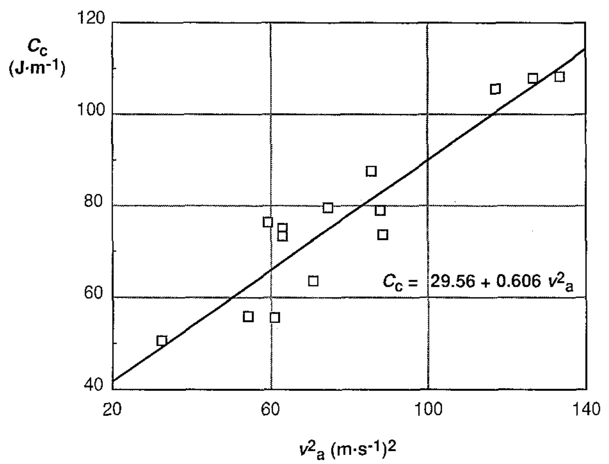

Fig. 2b. Energy cost of cycling for unit distance (C_c, J·m^{-1}) as a function of the square of air velocity (v_a^2, m^2·s^{-2}) — traditional frame, traditional wheels

(1974) for traditional racing bicycles who found values of 0.190 and 0.202 N·s^2·m^{-2}, respectively.

However, the contrary was the case in the present study in that the values of k obtained for aerodynamic bicycles with either lenticular or traditional wheels were significantly smaller than that obtained by di Prampero et al. (1979) for traditional frames ($P<0.05$). Thus, even though a direct comparison between the data obtained in the present study on traditional or aerodynamic frames could not be made for the reasons mentioned in the Results section, this provides good evidence that the aerodynamic frames were about 15% more economical than traditional ones in terms of mechanical energy dissipated against air resistance.

The constant k is a function of the area projected (A) of the air density (ρ) and of a dimensionless coefficient (c_d) which depends on the shape of the mobile object and which can be considered constant in the investigated range of speeds (Pugh 1973):

$$k = 0.5 \times c_d \times A \times \rho \tag{6}$$

The c_d can therefore be calculated from the above values of k, provided that A and ρ are constant and known. In this study, we determined A only for aerodynamic frames; it was equal to 0.395 m^2. Hence, taking k = 0.157 and $\rho = 1.233$ kg·m^{-3} (for $P_b = 758$ mmHg (101.1 kPa) and $T = 280.5$ K):

$$c_d = \frac{(2 \times k)}{(\rho \times A)} = \frac{0.314}{0.487} = 0.645 \tag{7}$$

i.e. approximately 18% less than the values reported by Pugh (1974) and by Kyle (1979) for conventional racing bicycles, which were approximately 0.80 in the dropped posture.

Ignoring for the sake of simplicity the minor effect of the water vapour, ρ is directly proportional to P_b and inversely proportional to the absolute T:

$$\rho = \rho_0 \times \left(\frac{P_b}{760}\right) \times \left(\frac{273}{T}\right) \tag{8}$$

where ρ_0 is the density of dry air at 273 K and 760 mmHg (101.3 kPa) (1.27 kg·m^{-3}). Therefore, from Eqs. 6 and 8, we obtained;

$$k = 0.5 \times c_d \times A \times \rho_0 \times 0.359 \times P_b/T \tag{9}$$

The mechanical power dissipated against wind ($\dot{W}_{air}$) is given by the product of the air resistance multiplied by the ground speed (v_g). Thus, inserting into Eq. 9 the values of c_d, A and ρ_0 as calculated and reported above, from Eq. 6 we obtained:

$$\dot{W}_{air} = k \times v_a^2 \times v_g = 5.81 \times 10^{-2} \times (P_b/T) \times v_a^2 \times v_g \tag{10}$$

where, obviously enough, in calm air $v_a = v_g$.

The constant R_r of Eq. 1 represents the energy losses due to the rotating parts of the bicycle and to the friction of the wheels with the terrain. The R_r is independent of the v_g, but depends substantially on the tyre pressures, on the characteristics of the tyres and of the terrain, and is proportional to the overall mass (subject plus bicycle). Divided by this last (expressed in newtons), R_r is a measure of the dimensionless rolling resistance coefficient (c_{rr}).

In the present experiment R_r is given by the R_t value applying for $v_a^2 = 0$ in Fig. 1a and b. Thus, as from Eqs. 1 and 2, it equals 0.35 and 2.4 N for AL and AT bicycles, respectively. Dividing these values by the total mass of the system (subject plus bike), c_{rr} can be calculated as 0.00042 and 0.0031. Whereas this last value of c_{rr} is of the same order as reported in the literature for smooth tracks (0.0021, Kyle 1986), that applying for AL bikes is far too low. Since the characteristics and tyre pressures were the same in both conditions, we are unable to explain this observation which may well have been due to the uncertainties involved in extrapolating the function of Fig. 1a to $v_a^2 = 0$. In all further calculations we will therefore assume a value of $R_r = 1.79$ N as calculated from the average data in the literature of c_{rr} for a mass of 87 kg (including the bike): $R_r = 0.0021 \times 83 \times 9.81 = 1.79$ N.

The overall mechanical power ($\dot{W}_t$) is given by the sum of the power dissipated against R_r plus $\dot{W}_{air}$. Since the latter has been given by Eq. 10 and since $R_r = 1.79$ N, the ($\dot{W}_t$) was given by:

$$\dot{W}_t = R_r \times v_g + D \times v_g = 1.79 \times v_g + 5.81 \times 10^{-2} (P_b/T) \times v_a^2 \times v_g \quad (11)$$

where, again, in calm air v_a equals v_g and D is drag. This equation shows also that the absolute value of R_r, which in this study was assumed to equal 1.79 N (see above) did not greatly affect the overall power output at high speeds. Indeed, for $R_r = 1.79$, it can be calculated from Eq. 11 that the power dissipated against R_r for speeds greater than 10 $m \cdot s^{-1}$ was less than 11% of the total.

For TL, the relationship between $\dot{W}_t$ and v_g was described by an equation similar to Eq. 11. In this case, however, the D component was 17.7% larger than that for AL bicycles (see Eqs. 2, 3; Table 3). Thus, assuming again $R_r = 1.79$ N and following the same line of reasoning as described above, we obtained:

$$\dot{W}_t = R_r \times v_g + D \times v_g = 1.79 \times v_g + 6.84 \times 10^{-2} (P_b/T) \times v_a^2 \times v_g \quad (11')$$

Energy expenditure

The overall metabolic energy expenditure above resting per unit time $\dot{M}_t$ is given by the product of Eqs. 3 and 4 times v_g:

$$\dot{M}_t = 30.8 \times v_g + 0.558 \times v_a^2 \times v_g \quad (12)$$

and

$$\dot{M}_t = 29.6 \times v_g + 0.606 \times v_a^2 \times v_g \quad (13)$$

for AT and TT bicycles, respectively, where $\dot{M}_t$ is given in watts and v_a and v_g in metres per second. The second terms of Eqs. 12 and 13 represent $\dot{M}_{air}$, which therefore was about 8% lower for aerodynamic than for traditional frames. Since $\dot{W}_{air}$ riding the aerodynamic bicycles was 17.7% (see above) less than riding the traditional frames, the aerodynamic bicycles would seem to be less economic in terms of metabolic than of mechanical energy output. In addition, the difference between the two coefficients of Eqs. 3 and 4 (Eqs. 12, 13) did not reach statistical significance. However, it should be pointed out that the difference in terms of energy expenditure between AT and TT bicycles was of the same order as that reported by McCole et al. (1990) who have shown that the total energy expenditure riding, cycles with aerodynamic frames at 11.1 $m \cdot s^{-1}$ was 7% ($\pm$4%) significantly less than with traditional frames.

The ratio of Eq. 11 to Eq. 12 yielded the overall mechanical efficiency of cycling (η) for AL bicycles which, for the present experimental condition values of P_b and T, increased asymptotically from 0.057 for v_a tending towards zero to 0.28 for v_a tending towards infinity. Thus, at speeds compatible with normal track cycling, but still in the aerobic range, η increased from 0.185 at 8.5 $m \cdot s^{-1}$ to 0.223 at 12.5 $m \cdot s^{-1}$, the mechanical power increasing from 112 to 329 W (see Eq. 11). In the same range of speeds, η for traditional frames (as given by the ratio of Eqs. 11′ to 13) increased from 0.207 to 0.247, up to 0.305 for v tending towards infinity. Both these data are consistent with those of Pugh (1974), Sargeant (1988) and Seabury et al. (1977) who have show that η increases with the exercise intensity. In addition, they showed that the aerodynamic racing bicycles, for a given mechanical power, are slightly less efficient than the conventional ones in terms of energy expenditure, possibly because of the constraints of the extremely bent forward position. Indeed, a similar observation has been previously reported by Welbergen and Clijsen (1990) who have found that the maximal mechanical power developed on a cycle-ergometer was significantly higher in the sitting than in the standard racing position. In contrast, maximal oxygen consumption was equal, thus suggesting that the mechanical efficiency of pedalling can indeed be affected by the posture of the cyclist.

Taking into account once again the effects of P and assuming that 1 ml $O_{2,STPD} = 20.9$ J at RQ = 0.96, Eqs. 12 and 13 can be rewritten as:

$$\dot{V}O_2 = 1.47 \times v_g + 0.988 \times 10^{-2} \times (P_b/T) \times v_a^2 \times v_g \quad (14)$$

and

$$\dot{V}O_2 = 1.42 \times v_g + 1.073 \times 10^{-2} \times (P_b/T) \times v_a^2 \times v_g \quad (15)$$

where $\dot{V}O_2$ is expressed in milliliters per second and v_a and v_g in metres per second. Hence $\dot{V}O_2$ can be calculated for any given v_g and v_a in cycling with aerodynamic or traditional frames on a flat track provided P_b and T are known.

Effects of body size

The effects of different body sizes on the energetics of cycling can be taken into account considering that the R_r of the equations of motion is proportional to the overall mass m_{tot}, (subject plus bicycle), whereas the mechanical of metabolic power dissipated against the wind ($\dot{W}_{air}$) are proportional to A of the mobile object. Neglecting A of the bicycle, it can be assumed that A is proportional to the subject's A_D. If this is so, Eqs. 11 to 15 can be modified as follows (see Tables 1, 2 for the actual values of m_{tot} and A_D:

$$\dot{W}_t = 0.0021 \times m_{tot} \times v_g + 2.99 \times 10^{-2} \times A_D \times (P_b/T) \times v_a^2 \times v_g \quad (16)$$

$$\dot{W}_t = 0.0021 \times m_{tot} \times v_g + 3.65 \times 10^{-2} \times A_D \times (P_b/T) \times v_a^2 \times v_g \quad (17)$$

and

$$\dot{V}O_2 = 1.8 \times 10^{-3} m_{tot} \times v_g + 5.08 \times 10^{-3} \times A_D \times (P_b/T) \times v_a^2 \times v_g \quad (18)$$

$$\dot{V}O_2 = 1.8 \times 10^{-3} m_{tot} \times v_g + 5.52 \times 10^{-3} \times A_D \times (P_b/T) \times v_a^2 \times v_g \quad (19)$$

These equations are a comprehensive quantitative description of the mechanics and energetics of track cycling with aerodynamic or traditional frames and lenticular wheels. They yield the mechanical power in watts and the energy requirement, expressed as oxygen consumption in millitres per second (1 ml $O_{2,STPD} \cdot s^{-1}$ = 20.9 W), as a function of v_a and v_g in metres per second and describe their dependence on P_b and T and on the body size of the subject.

In Eqs. 18 and 19, the area A was assumed to be proportional to the A_D. However, Swain et al. (1987) have shown that the $\dot{V}O_2$ per unit of body mass measured during cycling on flat terrain with traditional racing bicycles at a given speed is 22% lower in large than in small subjects ($P<0.01$). This is a finding that can be predicted from the fact that the larger subjects have a lower ratio A_D to body mass. However, the ratio of A_D to body mass of the larger cyclists of the Swain et al. (1987) study was only 11% lower than that of the small cyclists ($P<0.001$) whereas the A to body mass ratio was 16% lower in the larger cyclists ($P<0.001$). This shows that A is not really a fixed fraction of the individual A_D and that, in larger subjects, this coefficient is smaller than expected from the A_D to body mass ratio. Thus, since the $\dot{W}$ and $\dot{V}O_2$ in Eqs. 16–19 was set to be proportional to A_D, the mechanical power and energy requirements yielded by these equations at a given v_g, everything else being equal, could have been overestimated, especially in the larger subjects.

General discussion and conclusions

The aim of the present study was to assess R_t and C_c for subjects riding aerodynamic frame bicycles and to derive a general equation of motion.

The relationship between R_t or C_c and v_a^2 were obtained on two subjects only. However, since the phenomena investigated obey the laws of fluid mechanics, the above approach would seems to have been legitimate and sufficient for our purposes.

The present data supported and extended a previous analysis making possible to, assess quantitatively the effects of speed, P_b, ρ and body size on the mechanics and energetics of cycling on traditional racing bikes. In addition, the present study made possible the estimatation of the effects of aerodynamical frames on maximal performances as follows. As shown in the previous paragraphs, in calm air the metabolic power is a function of v_g as described by:

$$\dot{M}_t = R_r \times v_g + k \times v_g^3 \tag{20}$$

Assuming as a first approximation that a given subject under a given set of conditions can maintain the same maximal metabolic power riding traditional or aerodynamic bicycles, the following equation must apply:

$$R_{rT} \times v_{g,T} + k_T \times v_{g,T}^3 = R_{rA} \times v_{g,A} + k_A \times v_{g,A}^3 \tag{21}$$

where the suffixes T ands A designate traditional or aerodynamic bicycles, respectively. Since R_r can be assumed to be equal, and $v_{g,T}$ and $v_{g,A}$ are not very different from each other, the terms $R_{rT} \times v_{g,T}$ and $R_{rA} \times v_{g,A}$ are equal and can be cancelled. If this is so Eq. 21 becomes:

$$k_T \times v_{g,T}^3 \approx k_A \times v_{g,A}^3 \tag{22}$$

Rearranging Eq. 22 and inserting the value determined above for k_T and k_A (see Eqs. 4, 5):

$$\frac{v_{g,A}}{v_{g,T}} = \sqrt[3]{\frac{0.606}{0.558}} = 1.028 \tag{23}$$

So, for a given set of conditions, the increase of speed to be expected when riding aerodynamic versus traditional bikes is of the order of 3.0%. Unfortunately at present such a comparison cannot be made directly because no cyclist has ever attemptet two maximal performances under the same set of conditions, but riding the two different types of bicycle in question. However, comparison of the records obtained over the same distances before and after the appearance of the aerodynamic frames and wheels yields an average ratio of 1.0395 (±0.0234) (for 1.0 km to 20.0-km track events from a stationary start), which is close to the value calculated above.

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